Effect of interactions on quantized current pulses on a quantum Hall edge

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Motivation

- Experimental interest in having source of **single electrons/quasiparticles** [1].
- Using a linear sweep of a quantum dot, a **zero-noise pulse** can be created in the absence of Coulomb interactions [2].

Model of single particle source

Our model describes

- a **quantum Hall edge** of length L described as a Luttinger liquid
$$\hat{H}_0(t) = \frac{v_F}{2}\int_{-L/2}^{L/2}\frac{dx}{2\pi}\left(\partial_x\hat{\varphi}\right)^2, \qquad (1)$$
- a **quantum dot** with time-dependent energy $\varepsilon(t)$
$$\hat{H}_d(t) = \varepsilon(t)\hat{a}^\dagger\hat{a}, \qquad (2)$$
- **tunneling** $\lambda(t)$ between the edge and the dot
$$\hat{H}_{\text{tun}}(t) = \lambda(t)(\hat{\psi}^\dagger(0)\hat{a} + h.c.), \qquad (3)$$
- **Coulomb interactions** between the dot and the edge
$$\hat{H}_{\text{int}} = -\gamma\frac{g}{2\pi}\partial_x\hat{\varphi}(0)(\hat{a}^\dagger\hat{a} - \frac{1}{2}). \qquad (4)$$

The total Hamiltonian is
$$\Longrightarrow \hat{H}(t) = \hat{H}_0(t) + \hat{H}_d(t) + \hat{H}_{\text{tun}}(t) + \hat{H}_{\text{int}}. \quad (5)$$

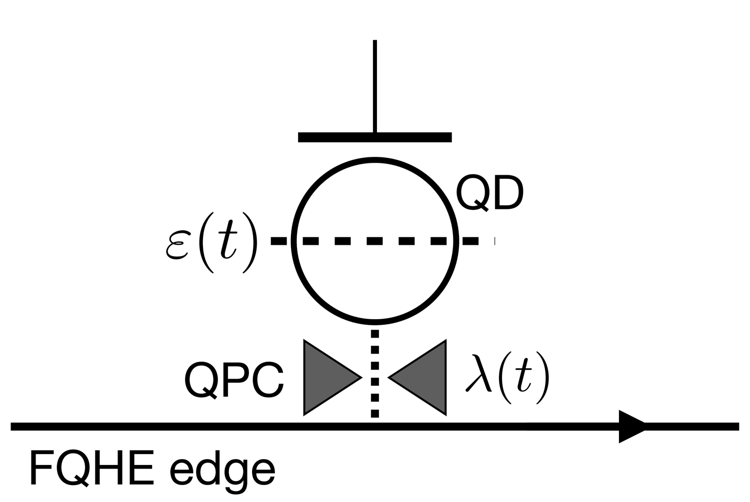

Figure: Schematic picture of the model.

Use **bosonization** identity
$$\hat{\psi}(x) = \frac{1}{\sqrt{2\pi}}a^{-\frac{\gamma^2}{2}}e^{-i\gamma\hat{\varphi}(x)}, \qquad (6)$$

- **electrons**: $\gamma = 1/\sqrt{\nu}$
- **quasiparticles(holes)**: $\gamma = \pm\sqrt{\nu}$

Mapping to the Spin-boson model

- Map Hamiltonian (5) to the spin-boson model.
- The spin-boson model is **well-studied** and there are **powerful numerical methods** for solving it, such as the **generalized master equation** approach.
- Use **unitary transformation** $\hat{U}_1 = e^{-i\gamma\hat{\varphi}(0)\hat{S}_z}$ suggested by Furusaki and Matveev [3].

Define spins
$$\hat{S}_z = \hat{a}^\dagger\hat{a} - \frac{1}{2}, \quad \hat{S}_x = \frac{1}{2}(\hat{a}^\dagger + \hat{a}), \quad \hat{S}_y = -\frac{i}{2}(\hat{a}^\dagger - \hat{a}). \qquad (7)$$

We arrive to the spin-boson Hamiltonian,
$$\hat{\tilde{H}} = \hat{U}_1^\dagger\hat{H}\hat{U}_1 = \varepsilon(t)\hat{S}_z + \Delta(t)\hat{S}_x + \sum_{k>0}\omega_k\hat{b}_k^\dagger\hat{b}_k - i\hat{S}_z\sum_{k>0}\eta_k(\hat{b}_k - \hat{b}_k^\dagger), \quad (8)$$
where $\omega_k = v_F k$ and $\Delta \propto \lambda$.

- Spin-1/2 in presence of a time-dependent magnetic field $\mathbf{B}(t) = \varepsilon(t)\mathbf{e}_z + \Delta(t)\mathbf{e}_x$.
- Bosonic heat bath (with Ohmic dissipation and dimensionless coupling $\alpha = \tilde{\gamma}^2/2$).
- Spin-boson coupling.

Current on the edge

- Measure current downstream from quantum point contact.
- The Hamiltonian can be **refermionized** and EOM yield current.

The current at $x > 0$ is given by
$$\hat{I}(x,t) = \tilde{q}\frac{d\hat{N}(t - \frac{x}{v_F})}{dt} \qquad (9)$$
and the current operator is defined as $\hat{I}(x,t) = v_F\hat{\rho}(x,t)$, where the charge density operator on the edge is $\hat{\rho}(x) = +e\sqrt{\nu}\partial_x\hat{\varphi}/2\pi$.

Renormalized charge

$$\tilde{q} = \left(1 - \frac{g}{2\pi v_F}\right)q. \qquad (10)$$

Numerics for current profile

Use the generalized master equation for $\alpha \ll 1$ and consider two sweep protocols.

Constant bias

- $\varepsilon(t) = \varepsilon_0$
- **separation of timescales**, the oscillations have frequency $\sim \Delta$ whereas the current decays with a rate $\ll \Delta$.

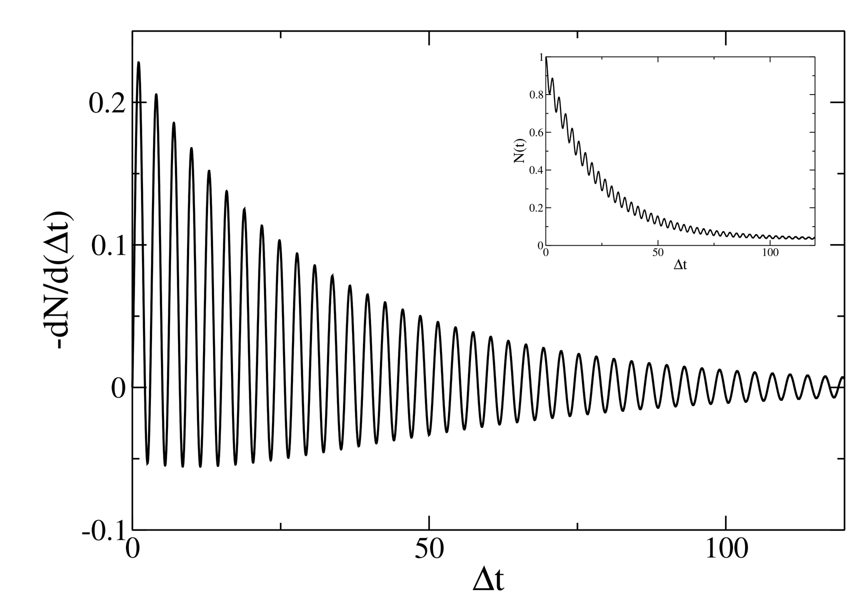

Figure: Current for constant bias.

Linear Landau-Zener sweep

- $\varepsilon(t) = \xi(t - t_0)$
- At late times the occupation of the quantum dot tends to $\exp(-\pi\Delta^2/2\xi)$.
- **Not all the charge leaks off the dot**, another impediment to quantized charge pulses.

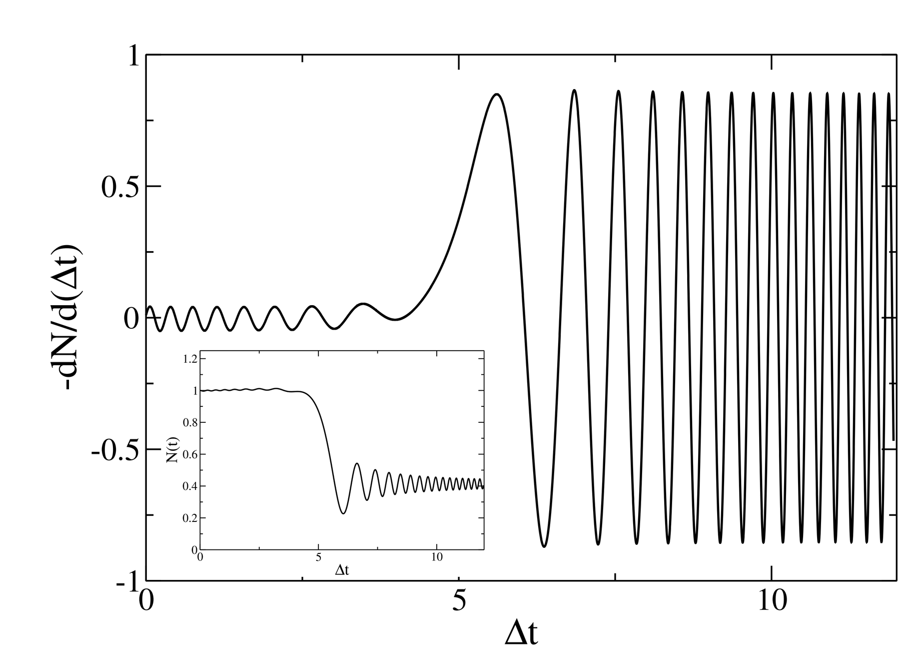

Figure: Current for Landau-Zener sweep.

Conclusions

- Coulomb interactions mean we **cannot obtain precisely quantized charge pulses**.
- The Coulomb interaction leads to a **depletion of the charge on the edge** around $x = 0$. When the quantized charge q leaves the dot, part of the quantized charge fills the depleted zone. Only a reduced charge $\tilde{q}$ is measured downstream.
- Mapping to spin-boson problem is useful, since it allows us to use **powerful numerical techniques** developed for solving this well-studied problem [4, 5].

References

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